Benha University
Faculty of Engineering- Shoubra
Electrical Engineering Department
First Year (Power)

Final Term Exam **Date:**
22 / 1 / 2015
Electrical and magnetic Properties of Materials
Total Duration : 3 hours

First Page

- Answer all the following question
- Illustrate your answers with sketches when necessary.
- The exam. Consists of two pages

- Mark of the 1st Page: **50** Mark
- Total Mark: **100** Marks
- 1st Page time: **1.5** hours

1.a) Write notes on each of the following items: (10 Marks)
1. Electrostrictive materials
2. Isotropic materials
3. The electric dipole moment
4. The total energy of the electron in different circular orbits
5. The internal electric field

1. b) Discuss the types of interaction between the particles of the microscopic domain. For polyatomic gas, find the relation between ε_r and temperature T. (7 Marks)

1. c) A gas has a number of molecules/cm^3= 2.5*10^{19}, its dielectric constant ε_r=1.0044 at a temperature 27 °c, the dipole moment of the molecules μ_p= 3.1*10^{-28} Coulombs.cm. Find the total polarizabilty of the molecules ($\alpha_e+\alpha_i$) and the orientational polarization for applied field E=1 Kvolt/cm.
If the temperature is decreased to 7 °c, find the corresponding value of ε_r . (8 Marks)

2.a) Derive an expression for the electronic polarizability $\overline{\alpha}_e$ in terms of ω.
Illustrate a schematic representation of the frequency dependence of the real and imaginary parts of the electronic polarizability (α_e', α_e'') for an atom contains one electron and then for n electrons. (12 Marks)

2.b) Find the rate of change of the orientational polarization of a liquid for the two cases:- (5 Marks)

1- Upon switching-on the field at t = 0
2- Upon switching-off the field at t = 0

2.c) Consider a parallel plate condenser filled with a dielectric between them characterized by a complex dielectric constant $\overline{\varepsilon_r} = \varepsilon_r' - j\,\varepsilon_r''$. The applied field $E_o \cos \omega t$, Find:

- An expression for the current density J(t)
- The dielectric losses per m^3 W(t).

Sketch the vector relationship between the field and the current. (8 Marks)

Good Luck &

Prof. Dr M. Moenes
Dr Tamer Elian

Please turn over the page

Model Answer for the Second Questions

2.a) Derive an expression for the electronic polarizability $\bar{\alpha}_e$ in terms of ω.
Illustrate a schematic representation of the frequency dependence of the real and imaginary parts of the electronic polarizability (α_e', α_e'') for an atom contains one electron and then for n electrons. (12 Marks)

The Answer:
Assume a nucleus of charge (+e) and a single electron, which can be represented by an electron cloud of a total charge (-e) distributed homogeneously through the volume of a sphere of radius R. The center of the sphere coincides with the nucleus in the absence of an external field.
When this atomic model is subjected to an alternating field, the following results would be obtained :-
1- The electron cloud will carry out the motion forced on it by the alternating field, since the nucleus is much heavier than the electron cloud.

2- The displacement (the shift) x of the electron cloud relative to the nucleus resulting from the field is much smaller than the electron cloud radius ($x << R$).
3- The restoring force F, which tends to derive the center of the cloud to the nucleus, i.e., to bring the cloud back to its original state, is given by

$$F = \frac{\left[\frac{(4/3)\pi x^3}{(4/3)\pi R^3} (-e) \right] (+e)}{4 \pi \varepsilon_o x^2}$$

Then,

$$F = \frac{-e^2}{4 \pi \varepsilon_o R^3} x \qquad \text{Then,} \qquad F = -a\, x \qquad \text{--------(1)}$$

Where,

$$a = \frac{e^2}{4 \pi \varepsilon_o R^3}$$

and a is called the restoring force constant.
The **equation of motion of the electron cloud** will be studied for the following three cases :-

1- In the absence of an applied field and with no damping

The equation of motion of the electron cloud is

$$m \frac{d^2 x}{d t^2} = -a\, x \qquad \text{---------(2)}$$

Where, m is the mass of the cloud, i.e., the electron mass.

The solution of this equation can be given by $x = x_o \sin(\omega_o t + \delta)$ ----------(3)
Where, x_o and δ are integration constants. Then,

$$\frac{dx}{dt} = x_o \omega_o \cos(\omega_o t + \delta)$$

and,

$$\frac{d^2x}{dt^2} = -x_o \omega_o^2 \sin(\omega_o t + \delta)$$

Substitute in Eq. (2), $m[-x_o \omega_o^2 \sin(\omega_o t + \delta)] = -a x_o \sin(\omega_o t + \delta)$
Then,

$$\omega_o^2 = \frac{a}{m}$$

or,

$$\omega_o = \sqrt{\frac{a}{m}}$$

Where, ω_o is called the natural or resonance angular frequency $\approx 10^{16}$ radian/second.
Substitute about a , then we can obtain,

$$\omega_o^2 = \frac{a}{m} = \frac{e^2}{4\pi\varepsilon_o m R^3}$$

In this equation, the only variable, which can affect the value of the natural frequency ω_o of each substance, is the radius of the electron cloud R.

2- In the absence of an applied field but the emission of electromagnetic radiation by the system may take into account.

The emission results from the time variation of the acceleration of the electron cloud and leads to damping. The damping due to radiation may be represented in the equation of motion of the electron cloud as follows

$$m\frac{d^2x}{dt^2} = -ax - 2b\frac{dx}{dt} \quad \text{---------(4)}$$

The constant b is related to the natural frequency ω_o by the following equation

$$2b = \frac{\mu_o e^2 \omega_o^2}{6\pi m c} \quad \text{------------(5)}$$

μ_o is the magnetic permeability of vacuum = $4\pi \times 10^{-7} = 1.257 \times 10^{-6}$ henry/meter
c is the speed of the light = 2.9979×10^8 meter/second

3- In the presence of an external alternating field

Let the field $\overline{E}$ be applied in the x direction and let it be represented by

$\overline{E} = E_o \cos \omega t$ Where, ω is being the angular frequency.

The force on the electron cloud resulting from the field $= (-e)\ E_o \cos \omega t$

The equation of motion will be as follows

$$m \frac{d^2 x}{d t^2} = - a x - 2 b \frac{d x}{d t} - e\ E_o \cos \omega t \qquad \text{-------------------}\mathbf{(6)}$$

To solve this equation for x(t), use the complex notation as shown

$$E_o \cos \omega t = R\ [E_o\ e^{j \omega t}] = E_o\ R\ [e^{j \omega t}] \qquad \text{--------(7)}$$

We shall assume the solution to be of the following form $\overline{x}(t) = R\ [\overline{A}\ e^{j \omega t}]$ -------(8)

Where, $\overline{A}$ is in general in complex amplitude. Then,

$$\frac{d x}{d t} = R\ [\overline{A}\ (j \omega)\ \mathbf{e}^{\mathbf{j \omega t}}\ \mathbf{]}$$

$$\frac{d^2 x}{d t^2} == R\ [\overline{A}\ (j \omega)^2\ \mathbf{e}^{\mathbf{j \omega t}}\ \mathbf{]}$$

Substitute the last two expressions into Eq.(4-6) to obtain,

$$R \left\{ \left[- \omega^2 \overline{A} + \frac{a}{m} \overline{A} + j \frac{2 b \omega}{m} \overline{A} + \frac{e}{m} E_o \right] e^{j \omega t} \right\} = 0$$

The expression in square brackets is zero, and write $(a/m) = \omega_o^2$, then

$$\overline{A} = \frac{\frac{e}{m} E_o}{\omega^2 - \omega_o^2 - j \frac{2 b \omega}{m}} \qquad \text{-----(9)}$$

The induced dipole moment as a function of time

In general, $\mu_{ind}(t) = -e\ x(t)$ Then, we can find from Eqs. (8) and (9)

$$\mu_{ind}(t) = R \frac{\frac{e^2}{m} E_o\, e^{j\omega t}}{\omega_o^2 - \omega^2 + j \frac{2 b \omega}{m}} \qquad \text{---(10)}$$

(4 Marks)

The static definition $\mu_{ind} = \alpha_e E$ can not be applied with the alternating field, therefore

$\mu_{ind}(t) = \mathbf{R}\left[\ \bar{\alpha}_e\ E_o\ e^{j\omega t}\ \right]$ ---(11) Where, $\bar{\alpha}_e$ is the complex polarizability.

$$\bar{\alpha}_e = \frac{\dfrac{e^2}{m}}{\omega_o^2 - \omega^2 + j\,\dfrac{2\,b\,\omega}{m}} \quad \text{-----(12)}$$

By writing out the real and imaginary parts of Eq. (12), we find

$$\bar{\alpha}_e = \frac{\dfrac{e^2}{m}\left(\omega_o^2 - \omega^2\right)}{\left(\omega_o^2 - \omega^2\right)^2 + \left(\dfrac{2\,b\,\omega}{m}\right)^2} - j\,\frac{\dfrac{e^2}{m}\,\dfrac{2\,b\,\omega}{m}}{\left(\omega_o^2 - \omega^2\right)^2 + \left(\dfrac{2\,b\,\omega}{m}\right)^2} \quad \text{-----(13)}$$

(4 Marks)

Then, $\bar{\alpha}_e = \alpha_e' - j\,\alpha_e''$

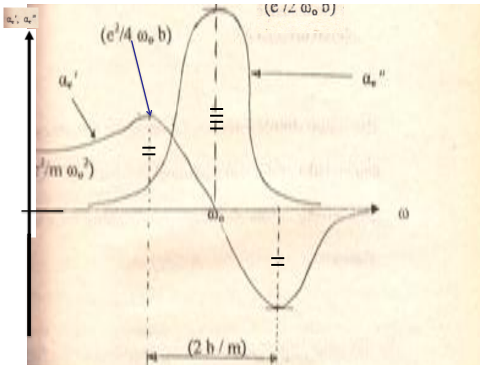
Where α_e' is the real part of the polarizability $\bar{\alpha}_e$, α_e'' is the imaginary part.

For $\omega = 0$, i.e., in static field :-

-The imaginary part will be vanished. - The real part will equal to the static value

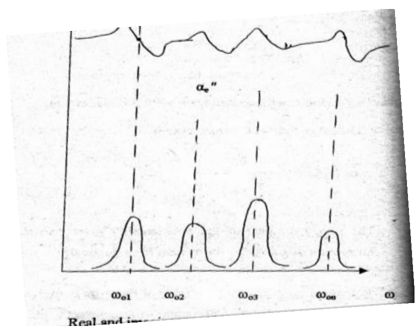
$$\alpha_e' \ (\text{at } \omega = 0) = \frac{e^2}{m\,\dfrac{a}{m}}$$

$$= \frac{e^2}{a} = \frac{e^2}{\dfrac{e^2}{4\pi\varepsilon_o R^3}} == 4\pi\varepsilon_o R^3$$



For one electron in the atom **(2 Marks)**

<u>For an atom contains a number of electrons n</u>, each of them corresponding to a particular force constant a_i and a particular damping constant b_i, the atom will have a series of ω_{oi} values and the polarizability will exhibit a frequency dependence as indicated in the following figure. **(2 Marks)**



2.b) Find the rate of change of the orientational polarization of a liquid for the two cases:- (5 Marks)

1- Upon switching-on the field at t = 0
2- Upon switching-off the field at t = 0

Answer:

The frequency dependence of the orientational polarization is of greater importance in liquids. Consider a liquid containing N permanent dipoles μ_p per unit volume.

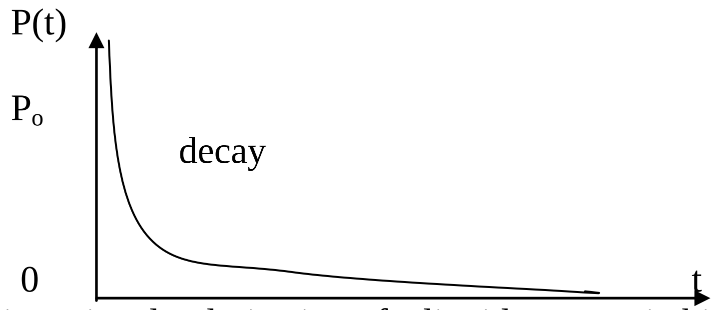
Suppose it has been subjected for a long time to a D.C. field E, let the orientational polarization in equilibrium with the field be P_o. When at the instant t = 0, the field is suddenly switched off, the polarization will not instantaneously become zero, because there is a certain time required for the rotation of the dipoles.

We shall assume that the polarization as function of time decays to zero in accordance with the formula $P_o(t) = P_o\, e^{-t/\tau}$

τ has the dimensions of time (The relaxation time) .

In a liquid, τ increases as the viscosity of the liquid increases. The rate of change of the polarization is evidently given by

$$\frac{d\,P_o(t)}{d\,t} = \frac{-P_o\, e^{-t/\tau}}{\tau} = \frac{-P_o(t)}{\tau}$$ (2.5 Marks)

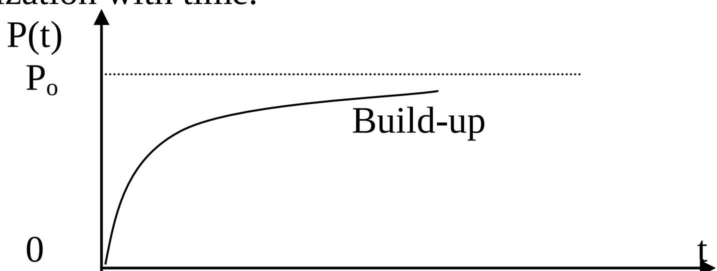


The decay of the orientational polarization of a liquid upon switching-off the field at t = 0.

- Suppose now that an external field has been absent for a long time, and that at t = 0 a field E is switched on. $P_o(t) = P_o(1 - e^{-t/\tau})$ During the build-up, the rate of increase is then

$$\frac{d P_o(t)}{dt} = \frac{P_o e^{-t/\tau}}{\tau} = \frac{[P_o - P_o(t)]}{\tau}$$

In the following figure, a field is switched on at t = 0, the curve represents the growth of the orientational polarization with time.



The growth of the orientational polarization with time. (2.5 Marks)

2.c) Consider a parallel plate condenser filled with a dielectric between them characterized by a complex dielectric constant $\overline{\varepsilon_r} = \varepsilon_r' - j\,\varepsilon_r''$. The applied field $E_o \cos \omega t$, Find:

- An expression for the current density J(t)
- The dielectric losses per m^3 W(t).

Sketch the vector relationship between the field and the current. (8 Marks)

Answer:

We have discussed the frequency dependence of the electronic, ionic and orientational contributions to the polarization. Since these contributions are additive, a material may be characterized by a complex dielectric constant $\overline{\varepsilon_r} = \varepsilon_r' - j\,\varepsilon_r''$ in which the real and imaginary parts ε_r' and ε_r'' incorporate all three contributions. The imaginary part gives rise to absorption of energy by the material from the alternating field. For this purpose consider a parallel plate condenser filled with a material characterized by $\overline{\varepsilon_r}$, the functions $\varepsilon_r'(\omega)$ and $\varepsilon_r''(\omega)$ are assumed to be given let the applied alternating voltage produce a field $E_o \cos \omega t$.

Suppose that at a given instant the charge per unit area on the plates is ±q(t). Since the flux density is numerically equal to the charge density, we must have D(t) = q(t) .

Also, since the current density is equal to

$J(t) = d\,q(t) / d\,t = d\,D(t) / d\,t$ amp / m^2 Since $E(t) = R\,[\,E_o\, e^{j\omega t}\,]$,
we may write in accordance with the meaning of the complex dielectric constant.

$$D(t) = R\,[\,\varepsilon_o\, \overline{\varepsilon_r}\; E_o\, e^{j\omega t}\,] = \varepsilon_o\, E_o\, R\,[\,\overline{\varepsilon_r}\; e^{j\omega t}\,]$$

Then, $J(t) = \varepsilon_o\, E_o\, R\,[\,(\varepsilon_r' - j\,\varepsilon_r'')\; j\,\omega\; e^{j\omega t}\,]$

$$= \omega\,\varepsilon_o\, E_o\,[\,\varepsilon_r''\; \cos\omega t\; - \varepsilon_r'\; \sin\omega t\,]$$

Since, $\sin\omega t = - j \cos\omega t$, then $J(t) = \omega\,\varepsilon_o\, E_o\,[\,\varepsilon_r''\; \cos\omega t + j\; \varepsilon_r'\; \cos\omega t\,]$

The imaginary part of the dielectric constant $\varepsilon_r''(\omega)$ determines the component of the current which is in phase with the applied field. Also, the real part of the dielectric constant $\varepsilon_r'(\omega)$ is coupled with a time factor which is 90 degrees out of phase with the applied field.

(3 Marks)

The instantaneous power per m^3 absorbed by the medium is given by J(t) E(t), hence, each second the material absorbs an amount of energy per m^3 given by

$$W(t) = \frac{1}{2\pi} \int_0^{2\Pi} J(t)\; E(t)\; d(\omega t)$$

Substituting J(t) , one readily finds

$$\mathbf{W(t)} = \frac{1}{2}\; \omega\; \varepsilon_o\; \varepsilon_r''\; E_o^{\,2}$$

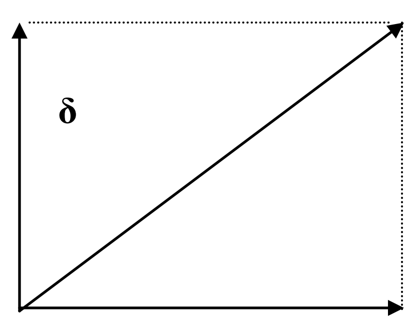
Thus, the absorption of energy is proportional to the imaginary part of the complex dielectric constant. Whenever there is energy dissipated in the medium we speak of dielectric losses.

A condenser containing a lossy dielectric may be represented by an equivalent circuit which consists of a pure capacitance and a parallel resistance, the latter being inversely proportional to $\varepsilon_r''\;\omega$. It is customary to characterize the losses of a dielectric at a certain frequency and temperature by the so-called " loss-tangent" **tan δ** , defined as

$$\tan\delta = \frac{\varepsilon_r''}{\varepsilon_r'}$$

The angle **δ** may be derived from that expression (3 Marks)

$\omega\,\varepsilon_o\,\varepsilon_r'\; E_o$ leads the field by 90 degrees.



Resultant current vector. (2 Marks)

$\omega\,\varepsilon_o\,\varepsilon_r''\; E_o$ in phase with the electric field

E_o →